ρ-Exchange Mechanism in the Reaction $\pi^+ + p \to N_{33}^{*++} + \pi^0$†

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We report a study of the reaction $\pi^+ + p \to N_{33}^{*++} + \pi^0$ at 3.54 BeV/c. We find very good agreement between the angular correlations observed in the decay of the N_{33}^* and the predictions by Stodolsky and Sakurai based on a single-ρ-exchange model. The reaction $K^+ + p \to N_{33}^{*++} + K^0$ has been found to display similar angular correlations, in agreement with the single-ρ-exchange mechanism. But we want to point out that the measured cross sections of both reactions decrease rapidly with increasing incident momentum, in contradiction with the results derived from a simple-ρ-exchange model as used, for example, by Jackson and Pilkuhn.

INTRODUCTION

THE reactions

$$\pi^+ + p \to N_{33}^{*++} + \pi^0 \quad (1)$$

and

$$K^+ + p \to N_{33}^{*++} + K^0 \quad (2)$$

are very suitable to study the ρ-meson-exchange mechanism. Parity conservation forbids the π (or η) exchange mode and isotopic-spin conservation forbids an $I=0$ meson-exchange mechanism (η or ω). Furthermore, the fact that the outgoing meson has no spin simplifies the theoretical predictions. Stodolsky and Sakurai,[1] who use a single-ρ-exchange model [Figs. 1(a) and 1(b)] in which the vertex $\rho^+ p N^*$ behaves like an $M1 \to p_{3/2}$ electromagnetic transition in accordance with the "ρ-photon analogy," have made predictions on the angular correlations in the decay of the N_{33}^{*++}. The reaction (2) has been found to display these predicted angular correlations at different energies.[2–4] We present here the study of the reaction (1) at 3.54 BeV/c. We also want to compare the measured cross section of both reactions (1) and (2) at different incident energies with the results derived from a single-ρ-exchange model as used, for example, by Jackson and Pilkuhn.[5]

EXPERIMENTAL PROCEDURE

After analyzing about 7000 two-prong events from a π^+ exposure at 3.54 BeV/c in the Brookhaven 20-in. hydrogen bubble chamber, we identified

$$2400 \text{ events as } \pi^+ + p \to \pi^+ + p, \quad (3)$$

$$1024 \text{ events as } \pi^+ + p \to \pi^+ + p + \pi^0, \quad (4)$$

$$732 \text{ events as } \pi^+ + p \to \pi^+ + \pi^+ + n. \quad (5)$$

The remaining events are mostly

$$\pi^+ + p \to \pi^+ + p + 2 \text{ or more neutral} \quad (6)$$

or

$$\pi^+ + p \to \pi^+ + \pi^+ + 2 \text{ or more neutral}. \quad (7)$$

The detail of the beam has been described elsewhere.[6]

We use the PACKAGE computer program (combination of PANG and KICK programs) for the track-reconstruction and fitting process. An event is considered to fit reaction (3) when it has $X^2 < 20.0$ for this reaction. It is considered to fit reaction (4) or (5) when it does not fit reaction (3) and when it has $X^2 < 5.5$ for either

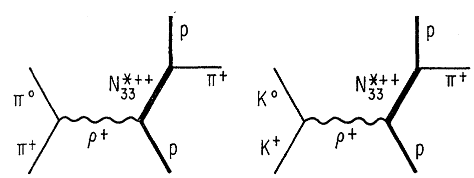

FIG. 1. ρ-exchange mechanism for isobar production: (a) for reaction $\pi^+ + p \to N_{33}^{*++} + \pi^0$, (b) for reaction $\pi^+ + p \to N_{33}^{*++} + K^0$.

reaction. A track ionization examination on the scanning table helped to resolve many (but not all) ambiguous events. In particular, about 5% of the reaction (4) events can also fit the reaction (5) and about another 5% of the reaction (4) events can be events from reaction (6).

Using a known total cross section[7] of 28.6±0.46 mb, we have deduced the partial cross sections for the 3 reactions at 3.54 BeV/c to be

$$\sigma(\pi^+ + p \to \pi^+ + p) = 4.86 \pm 0.5 \text{ mb},$$

$$\sigma(\pi^+ + p \to \pi^+ + p + \pi^0) = 2.05 \pm 0.2 \text{ mb},$$

$$\sigma(\pi^+ + p \to \pi^+ + \pi^+ + n) = 1.48 \pm 0.15 \text{ mb}.$$

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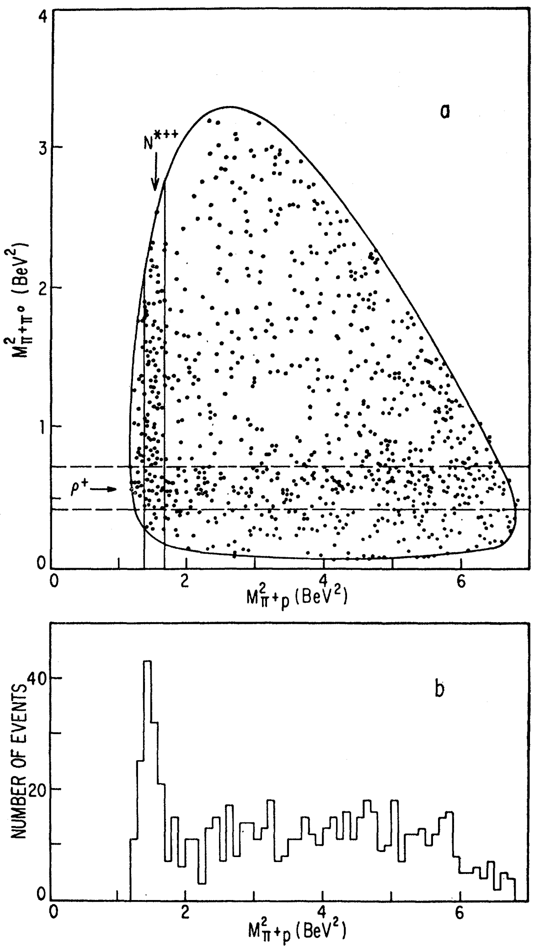

FIG. 2. (a) Dalitz plot of the reaction $\pi^+ + p \rightarrow \pi^+ + p + \pi^0$, (b) distribution of M_{π^2+p} from the reaction $\pi^+ p \rightarrow \pi^+ p \pi^0$ when $M_{\pi^+\pi^0}{}^2$ is outside the range of the ρ^+ meson ($M_{\pi^+\pi^0}{}^2 > 0.72$ BeV^2 and $M_{\pi^+\pi^0}{}^2 < 0.42$ BeV^2).

The Reaction $\pi^+ + p \rightarrow N_{33}{}^{*++} + \pi^0$ and the ρ-Exchange Mechanism

To show the existence of the reaction (1), we first plot, in Fig. 2(a), the Dalitz plot of the reaction (4). One can see the formation of ρ^+ around $M_{\pi^+\pi^0}{}^2 = 0.56$ BeV^2 and the formation of $N_{33}{}^{*++}$ around $M_{\pi^+p}{}^2 = 1.53$ BeV^2. In the Fig. 2 (b), we take off the ρ band (0.42 BeV$^2 < M_{\pi^+\pi^0}{}^2 < 0.72$ BeV^2) and plot the projection of the remaining Dalitz plot on the $M_{\pi^+p}{}^2$ axis. This histogram shows a big $N_{33}{}^{*++}$ peak above a small background. We estimate the cross section of the reaction (1) to be 0.20 ± 0.04 mb at 3.54 BeV/c.

In the following, we discuss only the reaction (1), i.e., we use only events with an effective mass M_{π^+p} in the $N_{33}{}^{*++}$. To avoid the overlap region of $N_{33}{}^{*++}$ and ρ^+, we analyzed the properties of the $N_{33}{}^{*++}$ resonance using the method of Eberhard and Pripstein,[8] whereby we ignored all the events on the Dalitz plot having 0.42 BeV$^2 < M_{\pi^+\pi^0}{}^2 < 0.72$ BeV^2, and repopulated this region with a sample of fictitious conjugated events constructed from the remaining events. (These fictitious events represent only 15% of the data.) We plotted in Fig. 3(a) the histogram of the angular distribution in $\cos\alpha$, where α is the angle between the normal to the plane of production and the direction of the π^+ from the $N_{33}{}^{*++}$ decay, measured in the $N_{33}{}^*$ rest frame. The shaded area represents the contribution of "fictitious" events. The solid curve represents the distribution "$1+3\cos^2\alpha$," normalized to the data. This is the prediction by Stodolsky and Sakurai[1] using a single-ρ-exchange model [Fig. 1(a)] in which the vertex $\rho^+ p N^*$ behaves like an $M1 \rightarrow p_{3/2}$ electromagnetic transition in accordance with the "ρ-photon analogy." Our data are in very good agreement with this prediction. We plot in Fig. 3(b) the histogram of the Treiman-Yang angle ϕ, where ϕ is the angle between the decay plane and the production plane in the $N_{33}{}^*$ rest frame. The shaded area is the contribution from "fictitious" events. The solid line represents the distribution "$1+2\sin^2\phi$" which can be derived from the preceding prediction ($1+3\cos^2\alpha$) assuming that the vertex $\rho p N^*$ is pure $M1 \rightarrow p_{3/2}$ transition. Here, also, our data agree very well with the prediction.

In Fig. 4, we plot the distribution of the differential cross section versus the four-momentum transfer ($d\sigma/d\Delta^2$) between the incident π^+ and the outgoing π^0. Our data peak sharply in the region of small momentum

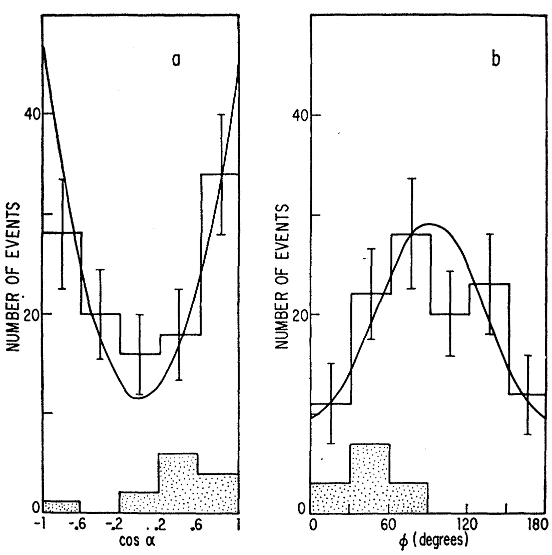

FIG. 3. (a) Angular distribution in $\cos\alpha$, the angle between the normal to the plane of production and the direction of the π^+ from $N_{33}{}^{*++}$ decay measured in the $N_{33}{}^{*++}$ rest frame for the reaction $\pi^+ + p \rightarrow N_{33}{}^{*++} + \pi^0$. The curve is $1+3\cos^2\alpha$, normalized to the data. (b) Histogram of the Treiman-Yang angle ϕ for the reaction $\pi^+ + p \rightarrow N_{33}{}^{*++} + \pi^0$. ϕ is the angle between the production plane and the decay plane of the $N_{33}{}^{*++}$ measured in the rest frame of $N_{33}{}^{*++}$. The curve is $1+2\sin^2\phi$ normalized to the data. In both (a) and (b) the shaded area is the contribution from fictitious events (see text).

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transfer. Jackson and Pilkuhn[5] have derived the differential production cross section of reactions (1) and (2) assuming a single-ρ-exchange mechanism with a magnetic dipole coupling model. Their formula is similar to the Stodolsky-Sakurai formula quoted by Daudin *et al.*[9] To make a fit to the momentum transfer distribution of reaction (2) at 3.0 BeV/*c* (which has almost the same total energy in the center of mass as in our case), they must use a form factor of the form $F_\rho(0)e^{-\lambda\Delta^2}$. They get good fits for

$$\lambda = 2.5 \quad \text{with} \quad (g_{\rho^+K^+K^0}{}^2/4\pi)(G_{\rho NN^*}{}^2/4\pi)|F_\rho(0)|^2 = 31$$

or

$$\lambda = 2.0 \quad \text{with} \quad (g_{\rho^+K^+K^0}{}^2/4\pi)(G_{\rho NN^*}{}^2/4\pi)|F_\rho(0)|^2 = 21.$$

Using their formula we get a good fit to our data (solid curve in Fig. 4) for

$$\lambda = 7.0 \quad \text{with} \quad (g_{\rho^+\pi^+\pi^0}{}^2/4\pi)(G_{\rho NN^*}{}^2/4\pi)|F_\rho(0)|^2 = 27.$$

By dividing the preceding relations, we get a ratio of $g_{\rho^+\pi^+\pi^0}{}^2/g_{\rho^+K^+K^0}{}^2$ between 0.9 and 1.3, which is not too far from 2.0, the predicted value of SU(3). But there are two criticisms of the results derived directly from ρ-exchange mechanism as with the Stodolsky-Sakurai formula[9] or with the Jackson-Pilkuhn formula[5]:

(a) One needs very drastic form factors to fit the data. The form factors are so rapidly varying that the momentum dependence of $1/(\Delta^2+M_\rho^2)$ is completely obscured [$e^{-7\Delta^2}$ is almost equivalent to $0.1/(\Delta^2+0.1)$, whereas $1/(\Delta^2+M_\rho^2)=1/(\Delta^2+0.56)$ with Δ^2 in BeV²]. This fact has been pointed out by many authors.[5,9]

(b) A stronger criticism is that the measured cross sections of both reactions (1) and (2) decrease rapidly with increasing incident momentum (see Table I) in contradiction with both the Stodolsky-Sakurai formula[9] and the Jackson-Pilkuhn formula[5] which predict an increasing cross section. In the Stodolsky-Sakurai formula[9] the cross section is almost proportional to

TABLE I. Cross section of reaction $\pi^+p \rightarrow N_{33}^{*++}+\pi^0$ and $K^+p \rightarrow N_{33}^{*++}+K^0$ at different incident momentum.

Reaction	Incident momentum in BeV/*c*	Cross section predicted by Jackson-Pilkuhn formula	Experimental cross section	Reference number
$\pi^+p \rightarrow N_{33}^{*++}+\pi^0$	3.54	0.16 mb	0.2±0.04 mb	This paper
	1.59	0.13 mb	1.5±0.2 mb	7
$K^+p \rightarrow N_{33}^{*++}+K^0$	3.0	0.8 mb	0.9±0.2 mb	4
	1.96	0.66 mb	2.7±0.4 mb	3
	1.14	0.28 mb	3.6±0.5 mb	a
	0.91	0.08 mb	2.1±0.2 mb	2

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[9] A. Daudin, M. A. Jabiol, C. Kochowski, C. Lewin, F. Selleri, S. Mongelli, A. Romano, and P. Waloschek, Phys. Letters **7**, 125 (1963). In this article the authors report on the reaction $\pi^++p \rightarrow N_{33}^{*++}+\pi^0$ at 1.59 BeV/*c*, but the agreement between their angular correlation distribution and the ρ-exchange prediction is inconclusive, due perhaps to limited statistics.

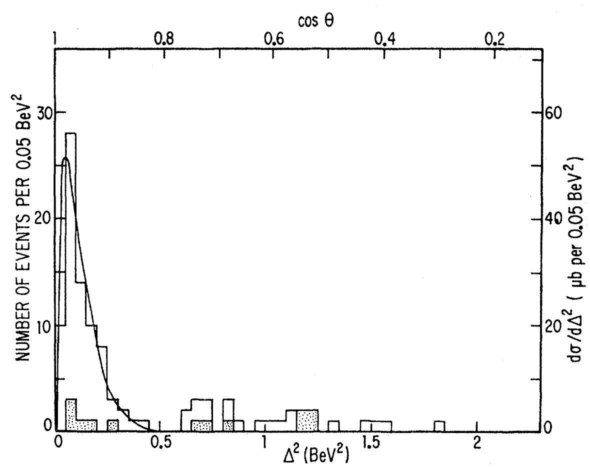

FIG. 4. Distribution of momentum transfer between incident π^+ and outgoing π^0 of the reaction $\pi^+p \rightarrow N_{33}^{*++}\pi^0$; the shaded area is the contribution from "fictitious" events (see text). On top we plot the scale in cosθ, where θ is the angle between π^0 and π^+ incident in the center-of-mass frame. The solid curve is the best fit using the "Jackson-Pilkuhn" formula (see text).

$P_i \cdot P_f{}^3$ with P_i= momentum of the incident π^+ and P_f = momentum of the outgoing π^0 in the center-of-mass frame. For the Jackson-Pilkuhn formula,[5] we list in Table I its predictions for reaction (1) at 3.54 BeV/*c* and 1.59 BeV/*c* (with λ=7.0), and its predictions for reaction (2) at 3.0 BeV/*c*, 1.96 BeV/*c*, 1.14 BeV/*c*, and 0.91 BeV/*c* (with λ=2.5). We also list the corresponding experimental results in the same table. Except for the momenta, where the calculations have been normalized to the data [3.54 BeV/*c* for reaction (1) and 3.0 BeV/*c* for reaction (2)], all the predicted cross sections are very different from the experimental values, sometimes by an order of magnitude.[10]

CONCLUSION

In summary, we would like to point out that while the angular correlations of the N^* decays of the reaction (1) and (2) agree very well with the predictions of the ρ-exchange model,[1] the production characteristics of both reactions (1) and (2) (momentum transfer distribution, energy dependence of the total cross sections) are in contradiction with a simple-ρ-exchange model.[5]

Ross and Shaw[11] have tried to improve the prediction of the one-pion-exchange mechanism by taking into account the absorption due to competing processes.

[10] We have verified that for all λ between 1.0 and 10.0, the Jackson-Pilkuhn formula gives values of cross section increasing with energy. One could argue that this formula is good only for small momentum transfer region and must therefore be smaller than the total cross section. Our answer is that most of the data are in the region of small momentum transfer and the difference would never make a factor of 10 observed sometimes between the predicted and measured cross section.

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Similar calculations must be done to improve the predictions of the "ρ-exchange model."

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Relation Between Masses of Pseudoscalar Octet and Vector Octet*

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The effect of the splitting of a pseudoscalar octet on the splitting of a vector octet is investigated using a simple effective-range formula for the coupling of a vector to two pseudoscalars. Taking the observed masses of the pseudoscalar octet to be $\eta(548)$, $K(496)$, and $\pi(140)$, it is found that the mass differences among the members of the vector octet φ, K^*, and ρ give the order $\varphi > K^* > \rho$.The magnitude of the calculated splitting is approximately twice the observed values. It is also shown that to first order in the mass splitting, if the pseudoscalar octet satisfies the Gell-Mann–Okubo mass formula, then the vector octet also satisfies the GMO formula. Furthermore, a deviation of the pseudoscalar masses from the GMO formula implies a somewhat larger deviation for the vector masses with an opposite sign. This result is in qualitative agreement with experimental values.

IT is well known that the octet of vector mesons ρ, K^*, and φ and the octet of pseudoscalar mesons π, K, and η do not satisfy the Gell-Mann–Okubo[1] mass formula exactly. In particular, the discrepancy for the vector mesons is larger than that for the pseudoscalar mesons. It is of some interest then to investigate the effect of the actual mass splitting of the pseudoscalar octet on the mass splitting of the vector octet[2] and, in particular, the consequences of the deviation of the pseudoscalar mesons from the GMO mass formula. Using the relativistic effective-range approximation for the coupling of a vector meson to two pseudoscalars, we show that taking the observed masses of the pseudoscalar octet to be $\eta(548)$, $K(496)$, and $\pi(140)$ the masses of the members of the vector octet are in the order $\varphi > K^* > \rho$. The magnitude of the calculated splitting is approximately twice the observed values. It is shown that to first order in the mass splitting, if the pseudoscalar octet satisfies the Gell-Mann–Okubo mass formula, then the vector octet also satisfies it. This is also true when the potentials are determined by the bootstrap mechanism. Furthermore, a deviation of the pseudoscalar masses from the GMO formula implies a larger deviation for the vector masses with the opposite sign. In other words, the $\eta(548)$ being lighter than the GMO prediction of 565 MeV implies that the φ should be heavier than the GMO prediction of 930 MeV. This result is in qualitative agreement with experimental values. However, as we shall see later, the violation of the GMO formula by the vector octet is substantially greater than the prediction derived from the first-order formula using observed values of η, K, and π.

Within the approximation of keeping only first-order terms in the mass differences, we examine several modifications of the effective-range formula. We find that these modifications do not change the result of the simple effective-range formula by more than 20 to 30%. On the other hand, in view of the large mass splitting within the pseudoscalar octet and the large value of the predicted first-order vector mass splitting, it appears that higher order terms in the mass differences could be quite important.

In the language of dispersion relations, the effective-range formula is a representation of the T matrix with the "potential" given by a simple pole in the unphysical region. In general, both the T matrix and the potential term are matrices of dimension equal to the number of channels having the same quantum numbers. For the

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